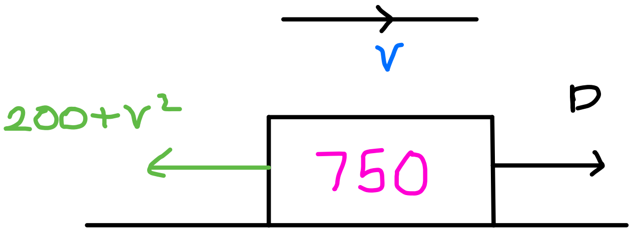
1) a) We need an equation linking velocity with force and power since velocity is unknown but the power and resistive force is given so we use the equation $P = Fv$. However we resolve first using $F = ma$. Also, since velocity is constant, the acceleration is zero.

$\therefore D - (200 + v^2) = 750(0)$ $\quad \therefore D = 200 + v^2$ $\quad \therefore$ We can substitute D into $P = Fv$ where the force (F) is the driving force (D).



$\therefore P = Fv$

$12000 = (200 + v^2) \times v$

$12000 = 200v + v^3$

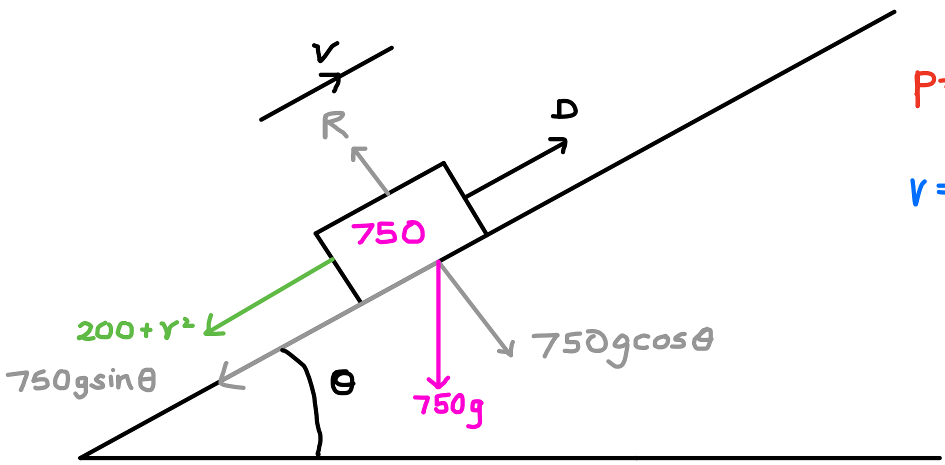
$v^3 + 200v - 12000 = 0$

Now we can either solve or substitute the equation.

$(20)^3 + 200(20) - 12000 = 8000 + 4000 - 12000 = 0$

$\therefore$ $v = 20\,ms^{-1}$ satisfies the equation.

b) Set up a diagram labelling all relevant forces.



$\sin\theta = \frac{1}{15}$

$P = 15kW = 15000W$

$v = 10 m/s$

To calculate the driving force we use the equation: $P = Fv$, where the force (F) is the driving force (D)

$\therefore P = Dv$

$\therefore D = \frac{P}{v} = \frac{15000}{10} = 1500N$

Now we use the formula: $\Sigma F = ma$ to find the acceleration.

$$\Sigma F = ma$$

$$D - (200 + v^2) - (750 g \sin\theta) = 750a$$

We can now substitute our values of D, v, g and $\sin\theta$ then rearrange for a.

$$1500 - (200 + (10)^2) - (750 \times 9.8 \times \tfrac{1}{15}) = 750a$$

$$1500 - 200 - 100 - 490 = 750a$$

$$710 = 750a \qquad a = \frac{710}{750} = 0.94666666\ldots$$

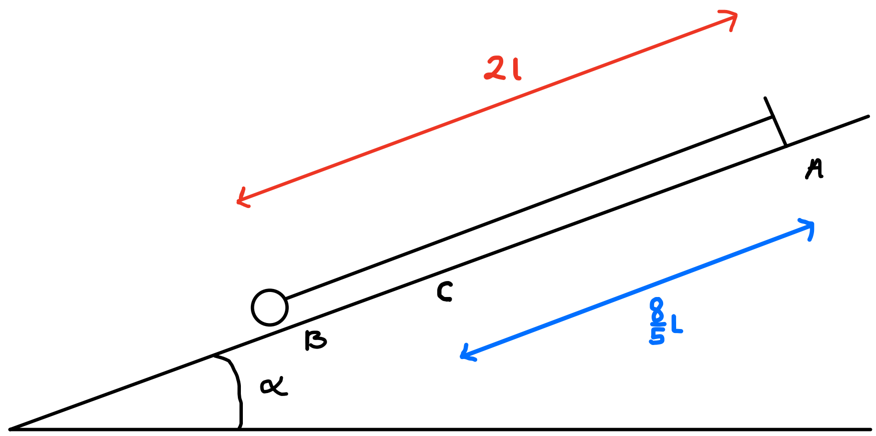
$$\approx 0.947\,ms^{-1} \ (3.s.f)$$

2) $\lambda = \frac{3}{4}mg$ $AB = 2L$

length $= L$ $AC = \frac{8}{5}L$

$\tan\alpha = \frac{5}{12}$

$\therefore \sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$



To calculate tension, we use the formula $T = \frac{\lambda e}{L}$.

$$\text{Tension} = \frac{\lambda e}{L} = \frac{(\frac{3}{4}mg)(2L - L)}{L} = \frac{3}{4}mg$$

We can now either draw a table showing the energies at the start and end or simply label on the energies on a before and after diagram.

Way 1:

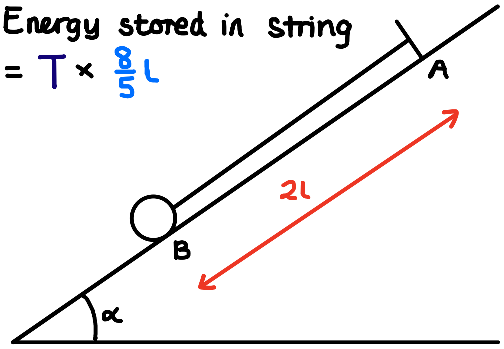
	GPE	EPE	Work Done by Friction
Start	0	$T \times \frac{8}{5}L$	0
End	$mg\sin\alpha \times T$	0	$-F_{max} \times T$ OR $-\mu mg\cos\alpha \times T$

F_{max} is negative as the work done opposes the motion hence the negative sign.

Way 2:

Energy stored in string

$= T \times \frac{8}{5}l$



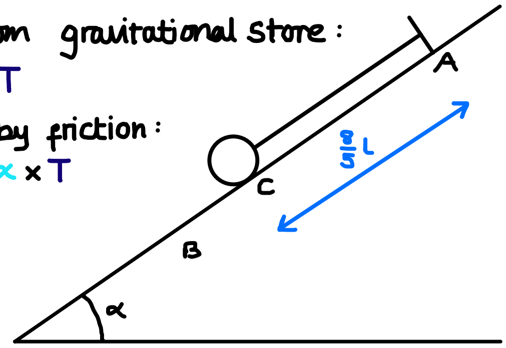
Start

Energy from gravitational store:

$= mg\sin\alpha \times T$

Work done by friction:

$= -\mu mg\cos\alpha \times T$



End

Using the Conservation of Energy Law meaning the energy at the start is conserved till the end meaning they are both equal. Therefore we can say:

Energy before = Energy after

$$T \times \frac{8}{5}l = mg\sin\alpha \times T + (-\mu mg\cos\alpha \times T)$$

Now we rearrange for μ after substituting in T.

$$\frac{3}{4}mg \times \frac{8}{5}l = mg\sin\alpha \times \frac{8}{5}l - \mu mg\cos\alpha \times \frac{8}{5}l$$

$$\therefore \; mg\sin\alpha \times \frac{8}{5}l - \mu mg\cos\alpha \times \frac{8}{5}l - \frac{3}{4}mg \times \frac{8}{5}l = 0$$

Here we can factorise $mg \times \frac{8}{5}l$ and also cancel it.

$$\therefore \; mg \times \frac{8}{5}l\left(\sin\alpha - \mu\cos\alpha - \frac{3}{4}\right) = 0 \qquad \therefore \; \sin\alpha - \mu\cos\alpha - \frac{3}{4} = 0$$

Now we substitute the values of $\sin\alpha$ and $\cos\alpha$ to solve for μ.

$$\frac{5}{13} - \mu\left(\frac{12}{13}\right) - \frac{3}{4} = 0$$

$$-\frac{19}{52} - \mu\left(\frac{12}{13}\right) = 0$$

$$\frac{12}{13}\mu = \frac{19}{52}$$

$$\therefore \; \mu = \frac{19}{52} \div \frac{12}{13} = \frac{247}{624} \approx 0.396 \text{ (3.s.f)}$$

3) a) To calculate impulse, we take away the initial momentum from the final momentum which essentially is the change in momentum where momentum is mass multiplied by velocity.

$$I = mv - mu$$

$$I = 3\begin{pmatrix}-1\\ \lambda\end{pmatrix} - 3\begin{pmatrix}2\\1\end{pmatrix}$$

$$I = \begin{pmatrix}-3\\ 3\lambda\end{pmatrix} - \begin{pmatrix}6\\3\end{pmatrix} = \begin{pmatrix}-3-6\\ 3\lambda-3\end{pmatrix} = \begin{pmatrix}-9\\ 3\lambda-3\end{pmatrix}$$

$$\therefore I = -9\underline{i} + (3\lambda-3)\underline{j}$$

Find the magnitude of the impulse using the $\underline{i}$ and $\underline{j}$ components:

$$|I| = \sqrt{(-9)^2 + (3\lambda-3)^2} = \sqrt{130}$$

$$\sqrt{81 + 9(\lambda-1)^2} = \sqrt{130}$$

$$81 + 9(\lambda-1)^2 = 130$$

$$9(\lambda-1)^2 = 49$$

$$(\lambda-1)^2 = \frac{49}{9}$$

$$\lambda - 1 = \pm\frac{7}{3}$$

$$\therefore \lambda = 1 \pm \frac{7}{3}$$

$$1 + \frac{7}{3} = \frac{10}{3} \qquad 1 - \frac{7}{3} = -\frac{4}{3}$$

Since λ is a positive constant,

$$\lambda = \frac{10}{3} \qquad \lambda \neq -\frac{4}{3}$$

$$\therefore I = -9\underline{i} + 3\left(\frac{10}{3} - 1\right)\underline{j}$$

$$= -9\underline{i} + 7\underline{j}$$

b) To find an angle between two vectors, we use the dot product :

$\dfrac{a \cdot b}{|a||b|} = \cos\theta$, which means we need to calculate $|a|$ and $|b|$ so :

$$|a| = \sqrt{2^2+1^2} = \sqrt{5} \qquad |b| = \sqrt{(-1)^2 + \left(\frac{10}{3}\right)^2} = \frac{\sqrt{109}}{3}$$

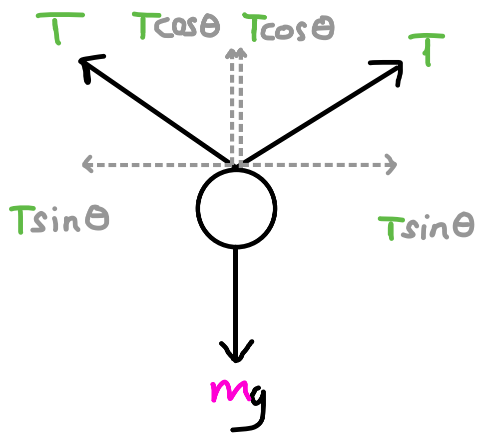
Therefore we can set up the equation and solve for θ where $\underline{a} = \begin{pmatrix}2\\1\end{pmatrix}$, $\underline{b} = \begin{pmatrix}-1\\10/3\end{pmatrix}$

$$\frac{\begin{pmatrix}2\\1\end{pmatrix} \cdot \begin{pmatrix}-1\\ \frac{10}{3}\end{pmatrix}}{\sqrt{2^2+1^2}\sqrt{(-1)^2+\left(\frac{10}{3}\right)^2}} = \cos\theta \qquad \frac{2(-1) + 1\left(\frac{10}{3}\right)}{\sqrt{5}\sqrt{\frac{109}{9}}} = \cos\theta$$

$$\therefore \cos\theta = \frac{4/3}{\sqrt{5}\,\frac{\sqrt{109}}{3}} = \frac{4}{\sqrt{5}\sqrt{109}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{4}{\sqrt{5}\sqrt{109}}\right) = 80.13419306 \approx 80.1^\circ \ (3.s.f)$$

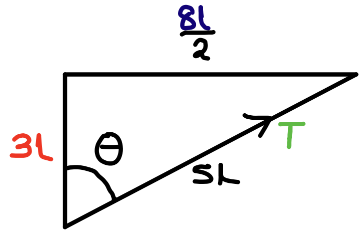
4) a) If in equilibrium, the sum of the vertical forces is equal to zero.



length = $8l$

mass = m

$\lambda = kmg$



$\therefore$ $2T\cos\theta - mg = 0$

$2T\cos\theta = mg$

$2T \times \frac{3}{5} = mg$

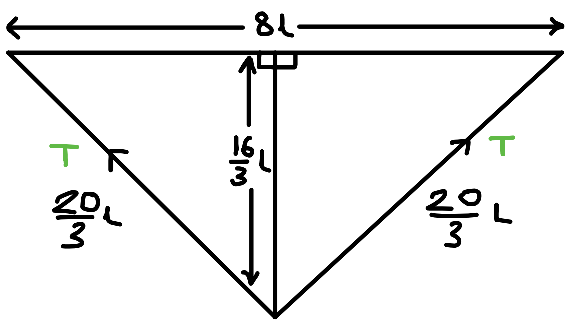
$\therefore$ $T = \frac{5}{6}mg$, using the fact that forces up is equal to the forces down.

We should also find T in terms of k to equate both and solve for k.

$$T = \frac{\lambda e}{L} = \frac{kmg \times (10l - 8l)}{8l} = \frac{kmg}{4} \qquad T = \frac{5}{6}mg$$

$$\therefore \frac{kmg}{4} = \frac{5}{6}mg \Rightarrow \frac{k}{4} = \frac{5}{6} \Rightarrow k = \frac{20}{6} = \frac{10}{3}$$

b) We should make a new diagram to account for the new extension.



$$\therefore \cos\theta = \frac{\frac{16}{3}L}{\frac{20}{3}L} = \frac{4}{5}$$

$$T = \frac{\frac{10}{3}mg \times \left(\frac{40}{3}L - 8L\right)}{8L} = \frac{\frac{10}{3}mg \times \frac{16}{3}L}{8L} = \frac{20}{9}mg$$

Now we use $F = ma$ to find the acceleration.

$\Sigma F = ma$

$2T\cos\theta - mg = ma$

$2 \times \frac{20}{9}mg\cos\theta - mg = ma$

$\frac{40}{9}g\cos\theta - g = a \qquad a = \frac{40}{9}(9.8) \times \frac{4}{5} - (9.8) = \frac{23}{9}g = \frac{1127}{45}\,ms^{-2}$

c) Extension = Extended length − Original length $= \frac{40}{3}L - 8L = \frac{16}{3}L$

The formula for elastic energy stored is $\frac{\lambda e^2}{2L}$.

Elastic energy stored $= \frac{\lambda e^2}{2 \times 8L} = \frac{\frac{10}{3}mg \times \left(\frac{16}{3}L\right)^2}{16L} = \frac{160}{27}mgL$ J

EPE used = Gain in GPE + Gain in KE

$$\therefore \frac{160}{27}mgL = mg\sqrt{\left(\frac{20}{3}L\right)^2 - (4L)^2} + \frac{1}{2}mv^2$$

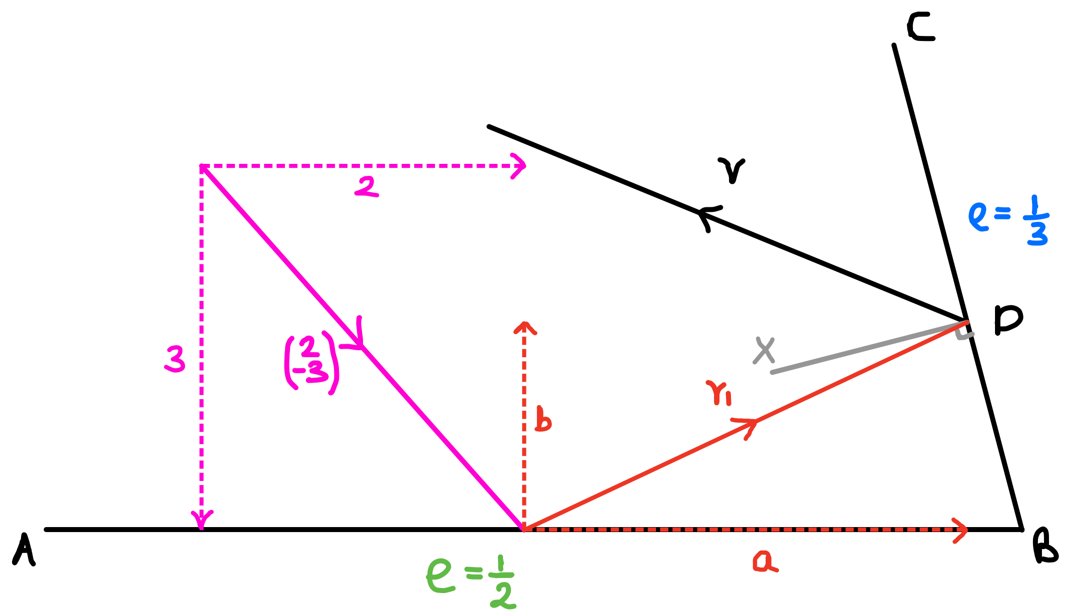
Now we rearrange for the velocity :

$$\frac{1}{2}mv^2 = \frac{160}{27}mgL - \frac{16}{3}mgL$$

$$\frac{1}{2}mv^2 = \frac{16}{27}mgL \qquad v^2 = \frac{32}{27}gL \qquad v = \sqrt{\frac{32}{27}gL} \text{ ms}^{-1}$$

d) The modulus of elasticity (λ) may be inaccurate.

5)



Let $v_1 \binom{a}{b}$ be velocity after impact with AB.

We also need to consider the point X to define the direction $\overrightarrow{DX}$ which is perpendicular to $\overrightarrow{BC}$ which allows us to break the velocity v_1 into components along each direction to account for the ball's motion in each direction after the impacts.

Its also not necessary to resolve v as we can calculate it directly using the rebounded components of v_1 in each direction and adding them together.

When a particle collides obliquely with a wall, the impulse acts perpendicular to the plane of impact meaning only the perpendicular components are affected (a) meaning the parallel component remains the same $\therefore a = 2$.

- We can use the value of e to find b.

$$e = \frac{\text{speed of seperation}}{\text{speed of approach}} = \frac{1}{2} = \frac{b}{3} \quad \therefore 2b = 3 \quad \therefore b = \frac{3}{2} \quad \therefore v_1 = \begin{pmatrix} 2 \\ 3/2 \end{pmatrix}$$

$\overrightarrow{BC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\quad \overrightarrow{DX} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ $\therefore$ the unit vectors for each vector are:

$$\hat{BC} = \frac{\overrightarrow{BC}}{|BC|} = \frac{\begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\sqrt{10}} \quad \text{and} \quad \hat{DX} = \frac{\overrightarrow{DX}}{|DX|} = \frac{\begin{pmatrix} -3 \\ -1 \end{pmatrix}}{\sqrt{10}}$$

Now we find the scalar components of v_1 in each of the directions:

$$\text{Component of } v_1 \text{ in direction } \overrightarrow{BC} = \frac{v_1 \cdot \overrightarrow{BC}}{|BC|} = \frac{\begin{pmatrix} 2 \\ 3/2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\sqrt{10}} = \frac{-2 + 9/2}{\sqrt{10}} = \frac{2.5}{\sqrt{10}}$$

$$\text{Component of } v_1 \text{ in direction } \overrightarrow{DX} = \frac{v_1 \cdot \overrightarrow{DX}}{|DX|} = \frac{\frac{1}{3}\begin{pmatrix} -2 \\ -3/2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -1 \end{pmatrix}}{\sqrt{10}} = \frac{\frac{1}{3}\left(6 + \frac{3}{2}\right)}{\sqrt{10}} = \frac{2.5}{\sqrt{10}}$$

This means the vector components of v_1 in each direction is:

$$\text{In direction } \overrightarrow{BC}: \quad \frac{2.5}{\sqrt{10}} \times \hat{BC} = \frac{2.5}{\sqrt{10}} \times \frac{1}{\sqrt{10}}\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \frac{2.5}{10}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

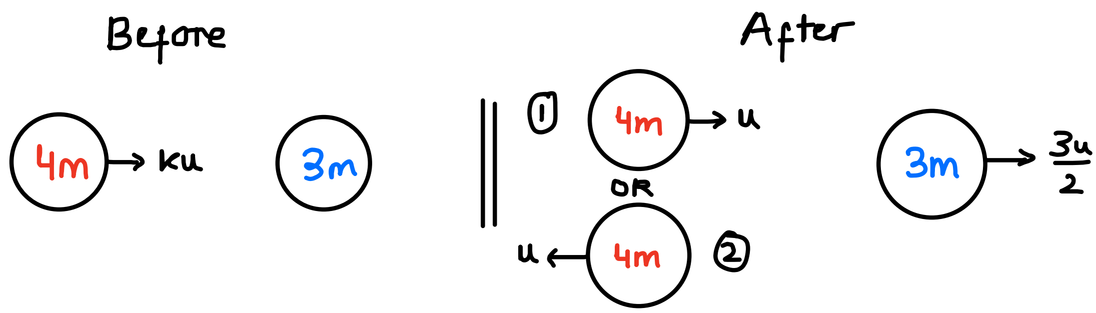
$$\text{In direction } \overrightarrow{DX}: \quad \frac{2.5}{\sqrt{10}} \times \hat{DX} = \frac{2.5}{\sqrt{10}} \times \frac{1}{\sqrt{10}}\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \frac{2.5}{10}\begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

Now we simply sum the vector components to find v because the total velocity is made up of it's parts in the directions of $\overrightarrow{BC}$ and $\overrightarrow{DX}$.

$$v = \frac{2.5}{10}\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \frac{2.5}{10}\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} -0.25 \\ 0.75 \end{pmatrix} + \begin{pmatrix} -0.75 \\ -0.25 \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{2} \end{pmatrix}$$

$$\therefore v = -\underline{i} + \tfrac{1}{2}\underline{j}$$

6) a) We can first draw a diagram of each balls' velocity before and after.



(i) Situation ① Using Conservation of Linear Momentum

$$4m \times ku + 3m \times 0 = 4m \times u + 3m \times \frac{3u}{2}$$

$$4k = 4 + \frac{9}{2} \qquad k = 1 + \frac{9}{8} = \frac{17}{8}$$

Situation ②

$$4m \times ku + 3m \times 0 = 4m \times -u + 3m \times \frac{3u}{2}$$

$$4k = -4 + \frac{9}{2} \qquad k = -1 + \frac{9}{8} = \frac{1}{8}$$

$$e = \frac{\text{speed of seperation}}{\text{speed of approach}} = \frac{\frac{3}{2}u - u}{\frac{1}{8}u} = 4$$

However, e cannot be greater than 1 $\therefore k \neq \frac{1}{8}$ $\therefore k = \frac{17}{8}$

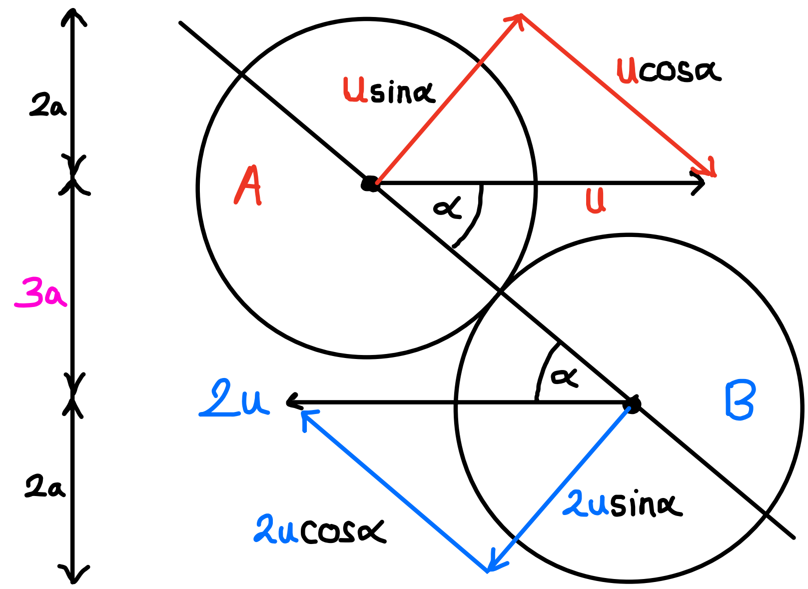
(ii) Since $k \neq \frac{1}{8}$, $k = \frac{17}{8}$ which means $e = \frac{\frac{3u}{2} - u}{\frac{17}{8}u} = \frac{4}{17}$

b) The loss in kinetic energy can be expressed by the initial kinetic energy minus the final kinetic energy.

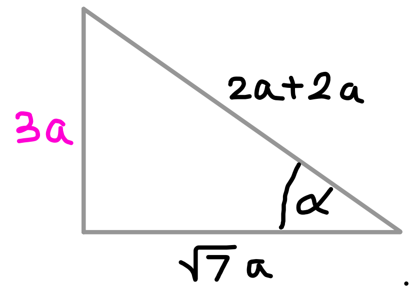
$$\therefore \text{Loss in KE} = \underbrace{\frac{1}{2}(4m)\left(\frac{17}{8}u^2\right)}_{\text{Initial KE}} - \underbrace{\left[\left(\frac{1}{2}(4m)(u^2)\right) + \left(\frac{1}{2}(3m)\left(\frac{3u}{2}\right)^2\right)\right]}_{\text{Final KE}}$$

$$= \frac{289}{32}mu^2 - \frac{43}{8}mu^2 = \frac{117}{32}mu^2 \text{ J}$$

7) a)



We can use trigonometry to find the distance between the line of centres but also sinα and cosα.



$\therefore \sin\alpha = \frac{3}{4}$

$\cos\alpha = \frac{\sqrt{7}}{4}$

Using Conservation of Linear Momentum along the line of centres:

$$3mu\cos\alpha - 2m \times 2u\cos\alpha = 3mV_A + 2mV_B$$

where V_A and V_B are velocity components along line of centres after collision.

$$e = \frac{1}{3} = \frac{V_B - V_A}{3u\cos\alpha}$$

$V_B - V_A = u\cos\alpha$

$V_B = V_A + u\cos\alpha$

Subsitute back into original equation to solve for V_A

$$3mu\cos\alpha - 2m \times 2u\cos\alpha = 3mV_A + 2mV_B$$

We can first collect like terms on the left hand side and cancel m.

$$\therefore \quad -u\cos\alpha = 3V_A + 2V_B$$

Now we simply substitute our value of V_B and rearrange for V_A.

$$-u\cos\alpha = 3V_A + 2(V_A + u\cos\alpha)$$

$$-u\cos\alpha = 5V_A + 2u\cos\alpha$$

$$5V_A = -3u\cos\alpha$$

$$V_A = -\frac{3}{5}u\cos\alpha$$

Impulse on A = Change in momentum along line of centres.
(Since there is no change in momentum perpendicular to line of centres)

$$\text{Impulse} = 3mV_A - 3mu\cos\alpha = 3m\left(-\frac{3}{5}u\cos\alpha\right) - 3mu\cos\alpha = -\frac{24}{5}mu\cos\alpha$$

$$= -\frac{24}{5}mu \times \frac{\sqrt{7}}{4} = -\frac{6\sqrt{7}}{5}mu \quad \therefore |I| = \left|-\frac{6\sqrt{7}}{5}mu\right| = \frac{6\sqrt{7}}{5}mu \text{ Ns}$$

b) Speed of A after the collision:

$$= \sqrt{V_A^2 + (u\sin\alpha)^2} = \sqrt{\left(-\frac{3u}{5}\times\frac{\sqrt{7}}{4}\right)^2 + \left(u \times \frac{3}{4}\right)^2}$$

$$= \sqrt{\frac{63u^2}{400} + \frac{9u^2}{16}} = \frac{3\sqrt{2}}{5}u \text{ ms}^{-1}$$

c) The impulse acts along the line of centres and not in any other direction.